Exercise 3.1

1. Formthepair of linear equations in thefollowing problems,and find their solutionsgraphically.

(i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost 50, whereas 7 pencils and 5 pens together cost 46. Find the cost of one pencil and that of one pen.

Solution:

(i)Let there are x number of girls and y number of boys. As per the given question, the algebraic expression can be represented as follows.

$x + y = 10$

$x - y = 4$

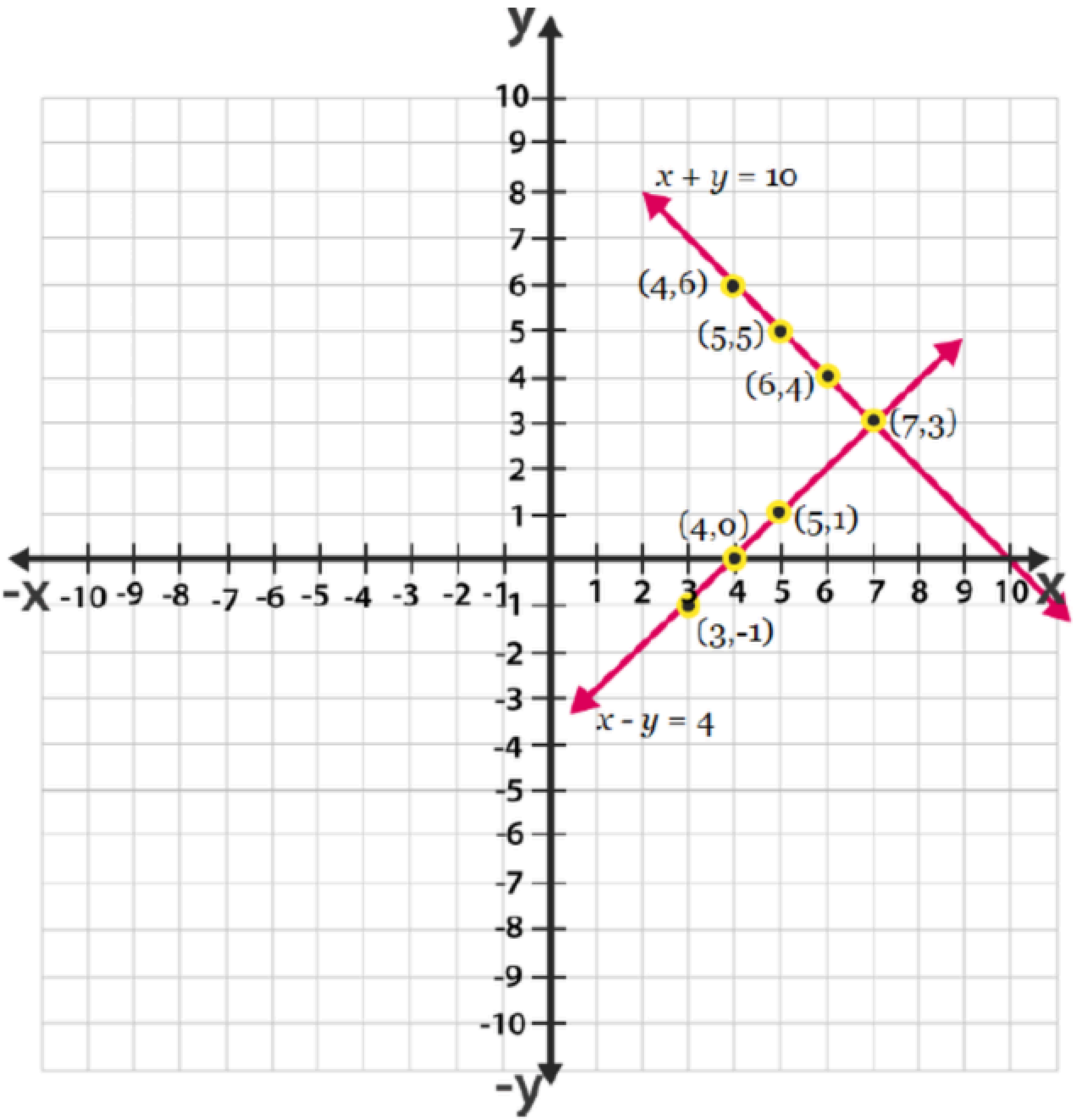
Now, for x+y = 10 or x = 10−y, the solutions are;

x	5	4	6
y	5	6	4

For x – y = 4 or x = 4 + y, the solutions are;

x	4	5	3
y	0	1	-1

The graphical representation is as follows;



From the graph, it can be seen that the given lines cross each other at point (7, 3). Therefore, there are 7 girls and 3 boys in the class.

(ii) Let1pencil costsRs.x and 1pencostsRs.y.

According to the question, the algebraic expression cab be represented as;

5x + 7y = 50

7x + 5y = 46

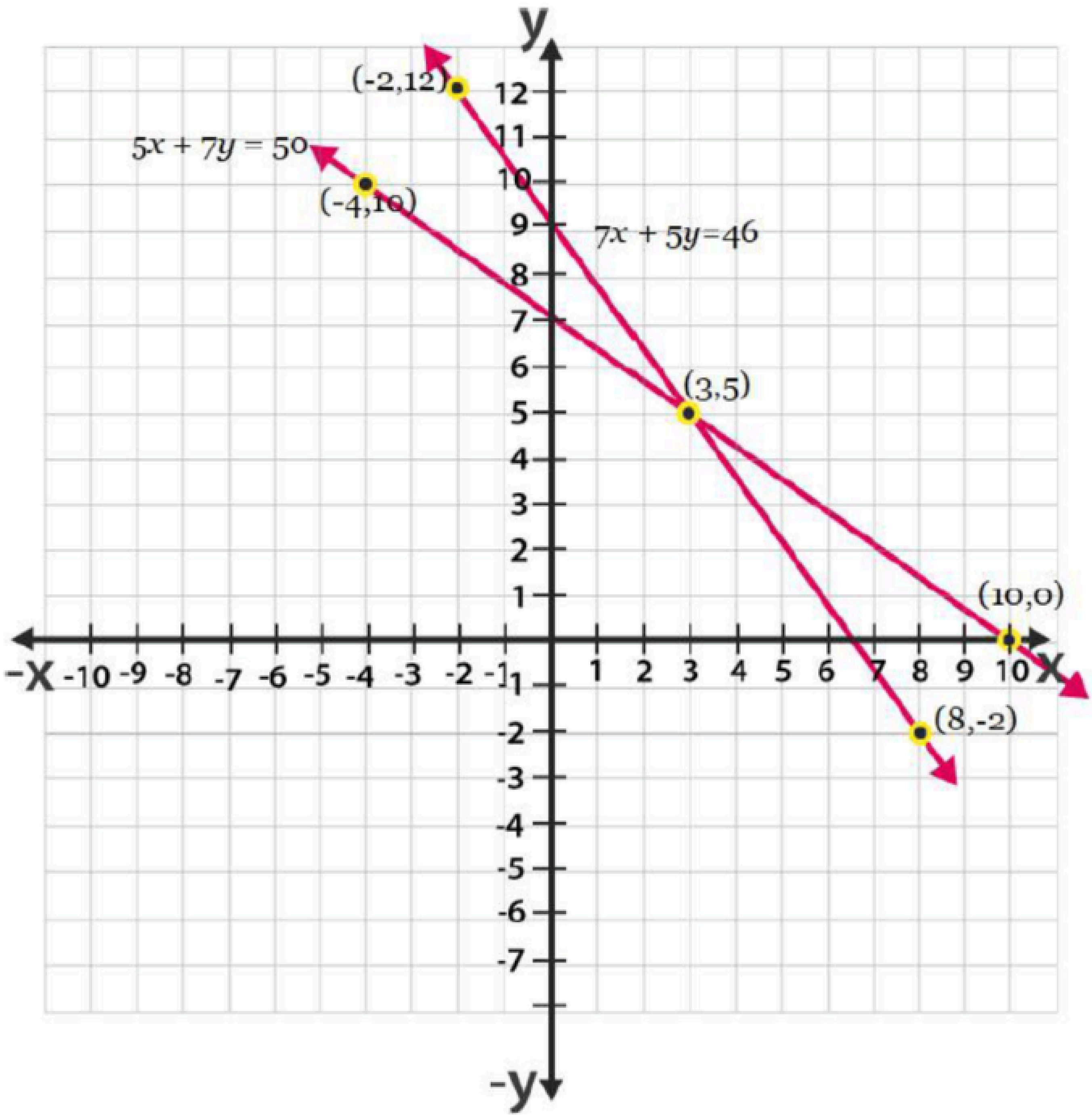
For, 5x + 7y = 50 or x = (50-7y)/5, the solutions are;

x	3	10	-4
y	5	0	10

For 7x + 5y = 46 or x = (46-5y)/7, the solutions are;

x	8	3	-2
y	-2	5	12

Hence, the graphical representation is as follows;



From the graph, it is can be seen that the given lines cross each other at point (3, 5).

So, the cost of a pencil is 3/- and cost of a pen is 5/-.

2. On comparing the ratios a1/a2 , b1/b2 , c1/c2 find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:

(i) 5x – 4y + 8 = 0

7x + 6y – 9 = 0

(ii) 9x + 3y + 12 = 0

18x + 6y + 24 = 0

(iii) 6x – 3y + 10 = 0

2x – y + 9 = 0

Solutions:

(i) Givenexpressions;

5x−4y+8=0

7x+6y−9=0

Comparingthese equations with a1x+b1y+c1 = 0

And a2x+b2y+c2 = 0

We get,

a1 = 5, b1 = -4, c1 = 8

a2 = 7, b2 = 6, c2 = -9

(a1/a2) = 5/7

(b1/b2) = -4/6 = -2/3

(c1/c2) = 8/-9

Since, (a1/a2) ≠ (b1/b2)

So, the pairs of equations given in the question have a unique solution and the lines cross each other at exactly one point.

(ii) Given expressions;

9x + 3y + 12 = 0

18x + 6y + 24 = 0

Comparingtheseequations with a1x+b1y+c1 = 0

And a2x+b2y+c2=0

We get,

a1 = 9,b1=3,c1=12

a2 = 18,b2=6,c2=24

(a1/a2) = 9/18 = 1/2

(b1/b2) = 3/6 = 1/2

(c1/c2) = 12/24 = 1/2

Since(a1/a2)=(b1/b2)=(c1/c2)

So, thepairsofequationsgiven in the question have infinite possible solutions and the lines are coincident.

(iii) GivenExpressions;

$6x – 3y + 10 = 0$

$2x – y + 9 = 0$

Comparing these equations with $a1x+b1y+c1 = 0$

And $a2x+b2y+c2 = 0$

We get,

$a1 = 6, b1 = -3, c1 = 10$

$a2 = 2, b2 = -1, c2 = 9$

$(a1/a2) = 6/2 = 3/1$

$(b1/b2) = -3/-1 = 3/1$

$(c1/c2) = 10/9$

Since $(a1/a2) = (b1/b2) \neq (c1/c2)$

So, the pairs of equations given in the question are parallel to each other and the lines never intersect each other at any point and there is no possible solution for the given pair of equations.

3. On comparing the ratio, (a1/a2) , (b1/b2) , (c1/c2) find out whether the following pair of linear equations are consistent, or inconsistent.

(i) $3x + 2y = 5$; $2x – 3y = 7$

(ii) $2x – 3y = 8$; $4x – 6y = 9$

(iii)$(3/2)x+(5/3)y = 7$; $9x – 10y = 14$

(iv) $5x – 3y = 11$; $– 10x + 6y = –22$

(v)$(4/3)x+2y = 8$; $2x + 3y = 12$

Solutions:

(i) Given : $3x + 2y = 5$ or $3x + 2y -5 = 0$

and $2x – 3y = 7$ or $2x – 3y -7 = 0$

Comparing these equations witha$1x+b1y+c1 = 0$

And $a2x+b2y+c2 = 0$

We get,

$a1 = 3, b1 = 2, c1 = -5$

$a2 = 2, b2 = -3, c2 = -7$

$(a1/a2) = 3/2$

$(b1/b2) = 2/-3$

$(c1/c2) = -5/-7 = 5/7$

Since, $(a1/a2) \neq (b1/b2)$

So, the given equations intersect each other at one point and they have only one possible solution. The equations are consistent.

(ii) Given 2x – 3y = 8 and 4x – 6y = 9

Therefore,

a1 = 2, b1=-3,c1=-8

a2 = 4, b2=-6,c2=-9

(a1/a2) =2/4=1/2

(b1/b2) =-3/-6=1/2

(c1/c2) =-8/-9=8/9

Since , (a1/a2)=(b1/b2) ≠ (c1/c2)

So, the equationsareparallel to each other andthey have no possible solution. Hence, the equations are inconsistent.

(iii)Given (3/2)x + (5/3)y = 7 and 9x – 10y = 14

Therefore,

a1 = 3/2,b1=5/3,c1=-7

a2 = 9,b2=-10,c2=-14

(a1/a2) = 3/(2×9) = 1/6

(b1/b2) = 5/(3× -10)= -1/6

(c1/c2) = -7/-14 = 1/2

Since, (a1/a2) ≠ (b1/b2)

So, theequationsareintersecting each other at one point and they have only one possible solution. Hence, the equations are consistent.

(iv) Given, 5x – 3y = 11 and – 10x + 6y = –22

Therefore,

a1 = 5,b1=-3,c1=-11

a2 = -10,b2=6,c2=22

(a1/a2)=5/(-10)=-5/10=-1/2

(b1/b2) = -3/6 = -1/2

(c1/c2) = -11/22 = -1/2

Since(a1/a2)=(b1/b2)=(c1/c2)

Theselinearequationsarecoincident lines and have infinite number of possible solutions. Hence, the equations are consistent.

(v)Given, (4/3)x +2y = 8 and 2x + 3y = 12

a1 = 4/3 , b1= 2 , c1 = -8

a2 = 2, b2 = 3 , c2 = -12

(a1/a2) = 4/(3×2)= 4/6 = 2/3

(b1/b2) = 2/3

(c1/c2) = -8/-12 = 2/3

Since (a1/a2) = (b1/b2) = (c1/c2)

These linear equations are coincident lines and have infinite number of possible solutions. Hence, the equations are consistent.

4. Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically:

(i) x + y = 5, 2x + 2y = 10

(ii) x – y = 8, 3x – 3y = 16

(iii) 2x + y – 6 = 0, 4x – 2y – 4 = 0

(iv) 2x – 2y – 2 = 0, 4x – 4y – 5 = 0

Solutions:

(i)Given, x + y = 5 and 2x + 2y = 10

(a1/a2) = 1/2

(b1/b2) = 1/2

(c1/c2) = 1/2

Since (a1/a2) = (b1/b2) = (c1/c2)

∴The equations are coincident and they have infinite number of possible solutions.

So, the equations are consistent.

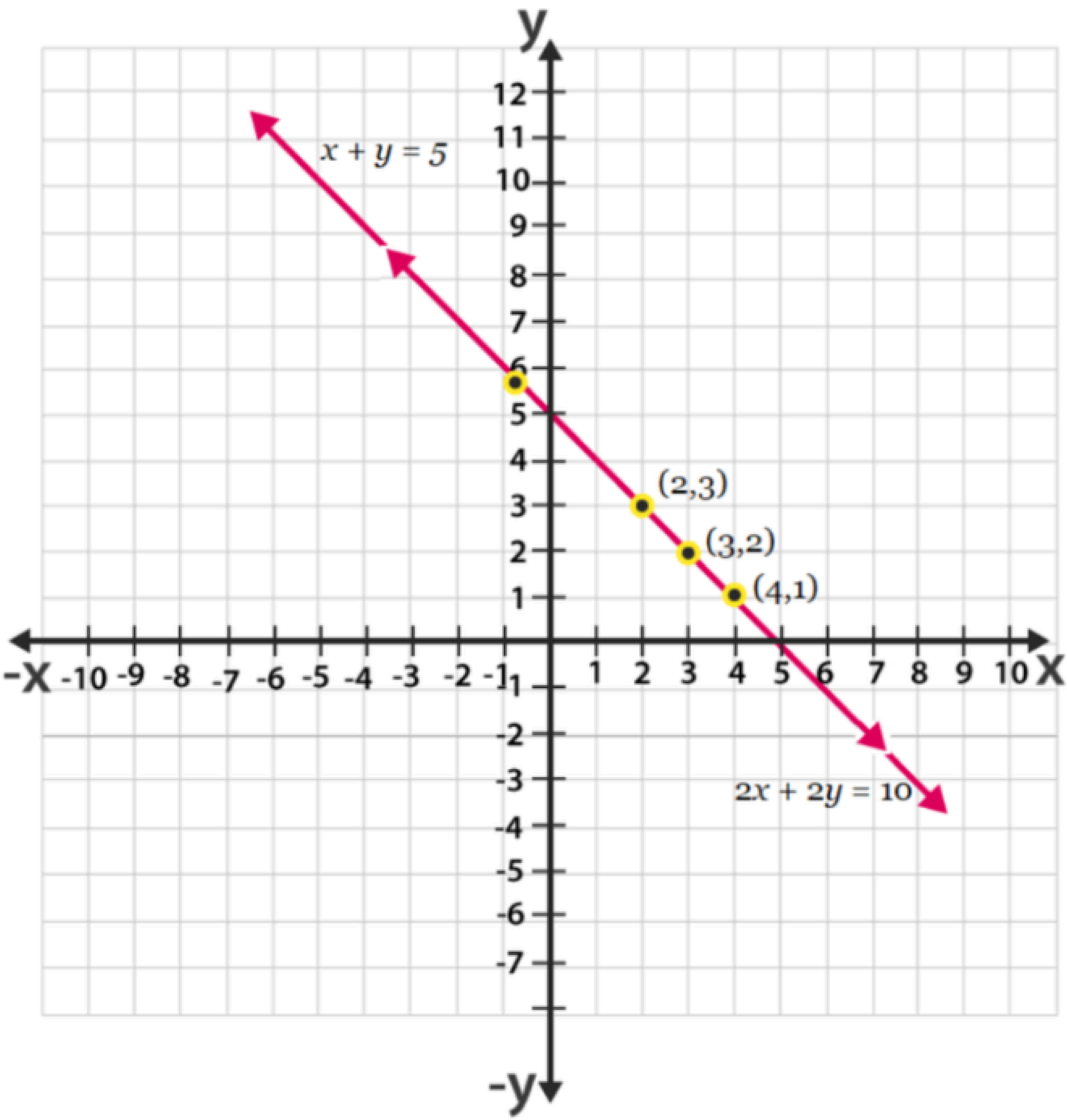
For, x + y = 5 or x = 5 – y

x	4	3	2
y	1	2	3

For 2x + 2y = 10 or x = (10-2y)/2

x	4	3	2
y	1	2	3

So, the equations are represented in graphs as follows:



From the figure, we can see, that thelines are overlapping each other.

Therefore, the equations have infinitepossible solutions.

(ii) Given, x – y = 8 and 3x – 3y = 16

(a1/a2) = 1/3

(b1/b2) = -1/-3 = 1/3

(c1/c2) = 8/16 = 1/2

Since, (a1/a2) = (b1/b2) ≠ (c1/c2)

The equations are parallel to each other and have no solutions. Hence, the pair of linear equations is inconsistent.

(iii) Given, 2x + y – 6 = 0 and 4x – 2y – 4 = 0

(a1/a2) = 2/4 = ½

(b1/b2) = 1/-2

(c1/c2) = -6/-4 = 3/2

Since, (a1/a2) ≠ (b1/b2)

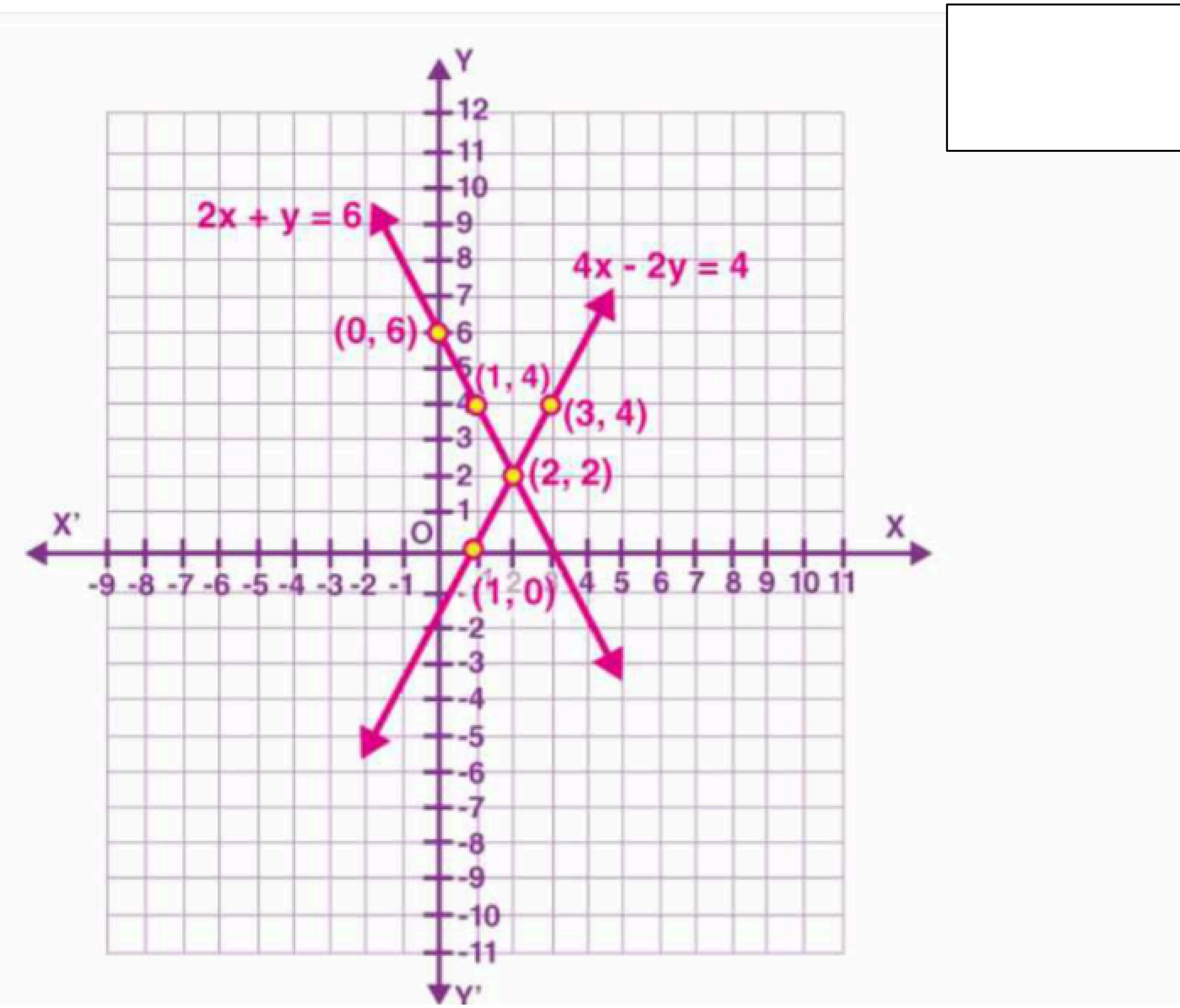
The given linear equations are intersecting each other at one point and have only one solution. Hence, the pair of linear equations is consistent.
Now, for ***2x + y – 6 = 0 or y = 6 – 2x***

x	0	1	2
y	6	4	2

And for ***4x – 2y – 4 = 0 or y = (4x-4)/2***

x	1	2	3
y	0	2	4

So, the equations are represented in graphs as follows:



From the graph, it can be seen that these linesare intersecting each other at only one point,(2,2).

(iv) Given, 2x – 2y – 2 = 0 and 4x – 4y – 5 = 0

(a1/a2) = 2/4 = ½

(b1/b2) = -2/-4 = 1/2

(c1/c2) = 2/5

Since, a1/a2 = b1/b2 ≠ c1/c2

Thus, these linear equations have parallel and have no possible solutions. Hence, the pair of linear equations are inconsistent.

5. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Solutions:Let us consider.

The width of the garden isxand length is y.

Now, according to the question, we can express the given condition as;

y – x = 4

and

y + x = 36

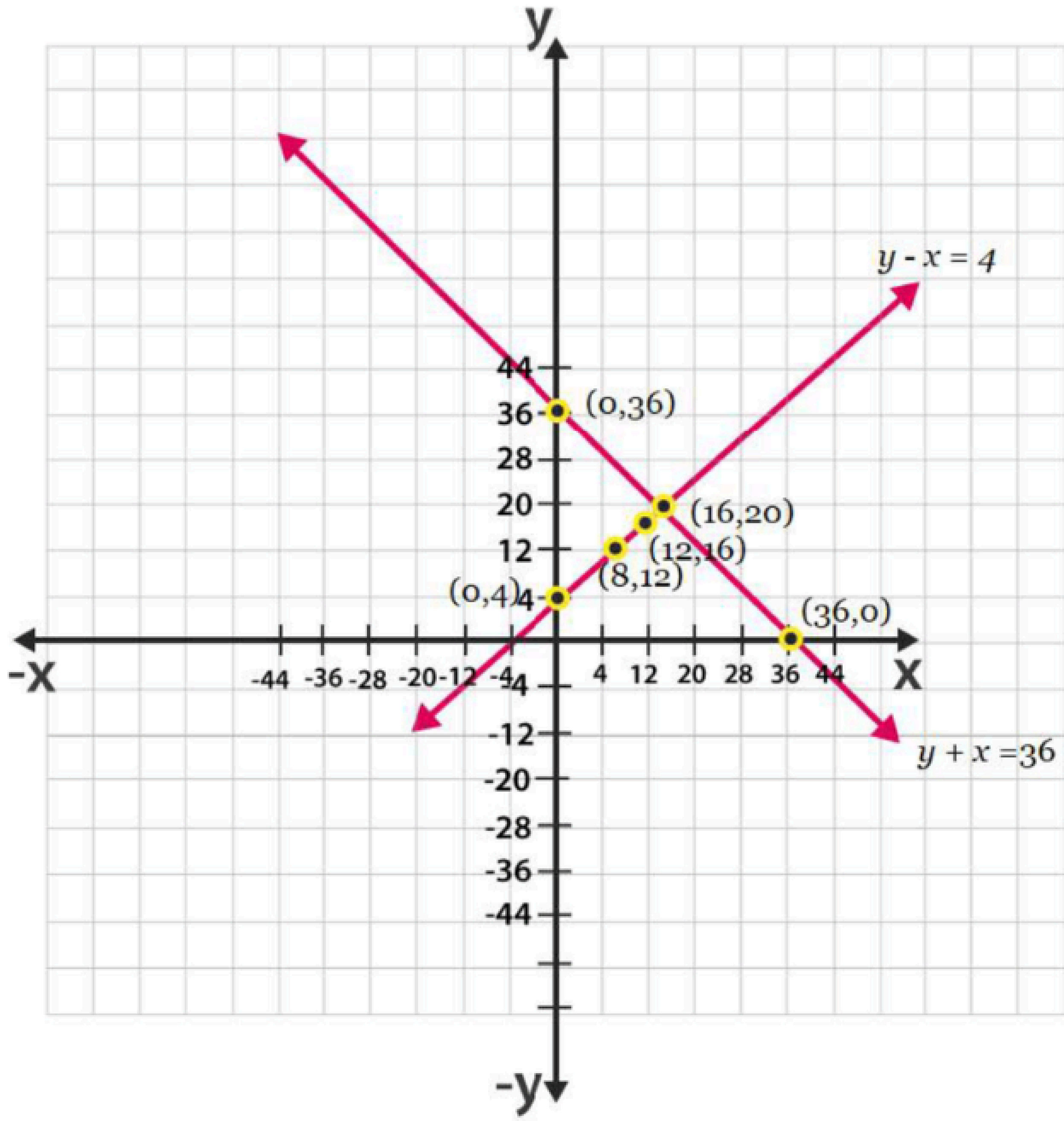
Now, taking y – x = 4 or y = x + 4

x	0	8	12
y	4	12	16

For y + x = 36, y = 36 – x

x	0	36	16
y	36	0	20

The graphical representation of both the equation is as follows;



From the graph you can see, the lines intersects each other at a point(16, 20). Hence, the width of the garden is16 and length is 20.

6. Given the linear equation $2x + 3y – 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:

(i) Intersecting lines

(ii) Parallel lines

(iii) Coincident lines

Solutions:

(i) Given the linear equation 2x + 3y – 8 = 0.

To find another linear equation in two variables such that the geometrical representation of the pair so formed is intersecting lines, it should satisfy below condition;

$(a1/a2) \neq (b1/b2)$

Thus, another equation could be $2x - 7y + 9 = 0$, such that;

$(a1/a2) = 2/2 = 1$ and $(b1/b2) = 3/-7$

Clearly, you can see another equation satisfiesthe condition.

(ii) Given the linear equation 2x + 3y – 8 = 0.

To find another linear equation in two variablessuch that the geometrical representation of the pair so formed is parallel lines, it should satisfy below condition;

$(a1/a2) = (b1/b2) \neq (c1/c2)$

Thus, another equation could be $6x + 9y + 9 = 0$, such that;

$(a1/a2) = 2/6 = 1/3$

$(b1/b2) = 3/9= 1/3$

$(c1/c2) = -8/9$

Clearly, you can see another equation satisfiesthe condition.

(iii) Given the linear equation 2x + 3y – 8 = 0.

To find another linear equation in two variablessuch that the geometrical representation of the pair so formed is coincident lines, it should satisfy below condition;

$(a1/a2) = (b1/b2) = (c1/c2)$

Thus, another equation could be $4x + 6y - 16 = 0$, such that;

$(a1/a2) = 2/4 = 1/2$,$(b1/b2) = 3/6 = 1/2$, $(c1/c2) = -8/-16 = 1/2$

Clearly, you can see another equation satisfies the condition.

7. Draw the graphs of the equations x – y + 1 = 0 and 3x + 2y – 12 = 0. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

Solution: Given, the equations for graphs are x – y + 1 = 0 and 3x + 2y – 12 = 0.

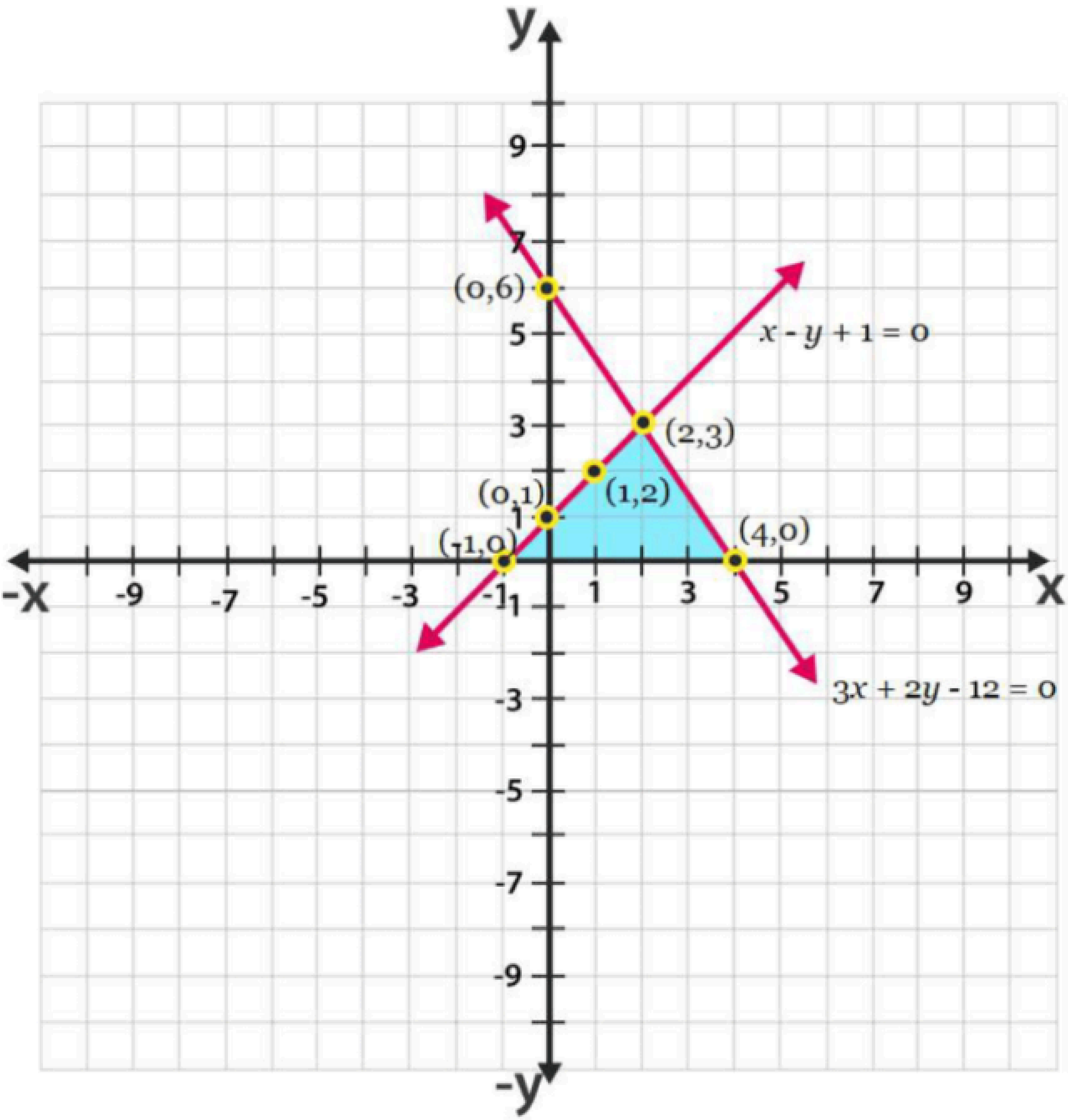
For, x – y + 1 = 0 or x = -1+y

x	0	1	2
y	1	2	3

For, 3x + 2y – 12 = 0 or x = (12-2y)/3

x	4	2	0
y	0	3	6

Hence, the graphical representation of these equations is as follows;



From the figure, it can be seen that these lines are intersecting each other at point (2, 3) and x-axis at (−1, 0) and (4,0). Therefore, the vertices of the triangle are (2, 3), (−1, 0), and (4, 0).

Exercise 3.2

1. Solve the followingpair of linearequationsbythesubstitutionmethod (i) x +

y = 14

x – y = 4

(ii) s – t = 3

(s/3) + (t/2) = 6

(iii) 3x – y = 3

9x – 3y = 9

(iv) 0.2x + 0.3y = 1.3

0.4x + 0.5y = 2.3

(v) √2 x+√3 y = 0

√3 x-√8 y = 0

(vi) (3x/2) – (5y/3) = -2

(x/3) + (y/2) = (13/6)

Solutions:

(i) Given,

x + y=14andx–y=4arethetwo equations.

From 1st equation, we get,

x =14–y

Now,substitutethevalueofxinsecond equation to get,

(14–y)–y=4

14–2y=4

2y = 10

Ory=5

Bythevalueofy,wecannowfind the exact value of x;

∵x=14–y

∴x=14–5

Orx=9

Hence, x = 9 and y = 5.

(ii) Given,

s –t=3and(s/3)+(t/2)=6arethe two equations.

From 1st equation, we get,

s = 3 + t ________________(1)

Now, substitute the value of s in second equation to get,

$(3+t)/3 + (t/2) = 6$

$\Rightarrow (2(3+t) + 3t)/6 = 6$

$\Rightarrow (6+2t+3t)/6 = 6$

$\Rightarrow (6+5t) = 36$

$\Rightarrow 5t = 30$

$\Rightarrow t = 6$

Now, substitute the value of t in equation (1)

$s = 3 + 6 = 9$

Therefore, s = 9 and t = 6.

(iii) Given,

$3x - y = 3$ and $9x - 3y = 9$ are the two equations.

From 1st equation, we get,

$x = (3+y)/3$

Now, substitute the value of x in the given second equation to get,

$9(3+y)/3 - 3y = 9$

$\Rightarrow 9 + 3y - 3y = 9$

$\Rightarrow 9 = 9$

Therefore, y has infinite values and since, $x = (3+y)/3$, so x also has infinite values.

(iv) Given,

$0.2x + 0.3y = 1.3$ and $0.4x + 0.5y = 2.3$are the two equations.

From 1st equation, we get,

x = (1.3- 0.3y)/0.2 __________________(1)

Now, substitute the value of x in the given second equation to get,

$0.4(1.3-0.3y)/0.2 + 0.5y = 2.3$

$\Rightarrow 2(1.3 - 0.3y) + 0.5y = 2.3$

$\Rightarrow 2.6 - 0.6y + 0.5y = 2.3$

$\Rightarrow 2.6 - 0.1y = 2.3$

$\Rightarrow 0.1y = 0.3$

$\Rightarrow y = 3$

Now, substitute the value of y in equation (1), we get,

x = (1.3-0.3(3))/0.2 = (1.3-0.9)/0.2 = 0.4/0.2 = 2

Therefore, x = 2 and y = 3.

(v) Given,

√2 x + √3 y = 0 and √3 x – √8 y = 0

are the two equations.

From 1st equation, we get,

x = – (√3/√2)y ____________________(1)

Putting the value of x in the given second equation to get,

√3(-√3/√2)y – √8y = 0 ⇒ (-3/√2)y- √8 y = 0

⇒ y = 0

Now, substitute the value of y in equation (1), we get,

x = 0

Therefore, x = 0 and y = 0.

(vi) Given,

(3x/2)-(5y/3) = -2 and (x/3) + (y/2) = 13/6 are the two equations.

From 1st equation, we get,

(3/2)x = -2 + (5y/3)

⇒ x = 2(-6+5y)/9 = (-12+10y)/9 ……………………….(1)

Putting the value of x in the given second equation to get,

((-12+10y)/9)/3 + y/2 = 13/6

⇒y/2 = 13/6 –((-12+10y)/27) + y/2 = 13/6

$$\frac{\frac{-12+10y}{9}}{3} + \frac{y}{2} = \frac{13}{6} \Rightarrow \frac{-12+10y}{27} + \frac{y}{2} = \frac{13}{6}$$

$$\Rightarrow \frac{y}{2} = \frac{13}{6} - \frac{-12+10y}{27} \Rightarrow \frac{y}{2} = \frac{117}{54} - \frac{-24+20y}{54}$$

$$\Rightarrow \frac{y}{2} = \frac{117+24-20y}{54}$$

$$\Rightarrow y = 3$$

Now, substitute the value of y in equation (1), weget,

(3x/2) – 5(3)/3 = -2

⇒ (3x/2) – 5 = -2

⇒ x = 2

Therefore, x = 2 and y = 3.

2. Solve 2x + 3y = 11 and 2x – 4y = – 24 and hence find the value of 'm' for which y = mx + 3.

Solution:

2x + 3y =11…………………………..(I)

2x – 4y =-24………………………… (II)

From equation (II), we get

x = (11-3y)/2 ………………….(III)

Substituting the value of x in equation (II), we get

2(11-3y)/2 – 4y = 24

11 – 3y – 4y = -24

-7y = -35

y = 5……………………………………..(IV)

Putting the value of y in equation (III), we get

x = (11-3×5)/2 = -4/2 = -2

Hence, x = -2, y = 5

Also,

y = mx + 3

5 = -2m +3

-2m = 2

m = -1

Therefore the value of m is -1.

3. Form the pair of linear equations for the following problems and find their solution by substitution method.

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

Solution:

Let the two numbers be x and y respectively, such that y > x.

According to the question,

y = 3x ……………… (1)

y – x = 26 …………..(2)

Substituting the value of (1) into (2), we get

3x – x = 26

x = 13 ……………. (3)

Substituting (3) in (1), we get y = 39

Hence, the numbers are 13 and 39.

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

Solution:

Let the larger angle by xo and smaller angle be yo.

We know that the sum of two supplementary pair of angles is always 180o.

According to the question,

x + y = 180o……………. (1)

x – y = 18o ……………..(2)

From (1), we get x = 180o – y …………. (3)

Substituting (3) in (2), we get

180o – y – y =18o

162o = 2y

y = 81o ………….. (4)

Using the value of y in (3), we get

x = 180o – 81o

= 99o

Hence, the angles are 99o and 81o.

(iii) The coach of a cricket team buys 7 bats and 6 balls for Rs.3800. Later, she buys 3 bats and 5 balls for Rs.1750. Find the cost of each bat and each ball.

Solution:

Let the cost a bat be x and cost of a ball be y.

According to the question,

7x + 6y = 3800 ………………. (I)

3x + 5y = 1750 ………………. (II)

From (I), we get

y = (3800-7x)/6………………..(III)

Substituting (III) in (II). we get,

3x+5(3800-7x)/6 =1750

⇒3x+ 9500/3 – 35x/6 = 1750

⇒3x- 35x/6 = 1750 – 9500/3

⇒(18x-35x)/6 = (5250 – 9500)/3

⇒-17x/6 = -4250/3

⇒-17x = -8500

x = 500 ……………………….. (IV)

Substituting the value of x in (III), we get

$y = (3800-7 \times 500)/6 = 300/6 = 50$

Hence, the cost of a bat is Rs 500 and cost of a ball is Rs 50.

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is Rs 105 and for a journey of 15 km, the charge paid is Rs 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?

Solution:

Let the fixed charge be Rs x and per km charge be Rs y.

According to the question,

$x + 10y = 105$ …………….. (1)

$x + 15y = 155$ …………….. (2)

From (1), we get $x = 105 – 10y$ ………………. (3)

Substituting the value of x in (2), we get

$105 – 10y + 15y = 155$

$5y = 50$

$y = 10$ …………….. (4)

Putting the value of y in (3), we get

$x = 105 – 10 \times 10 = 5$

Hence, fixed charge is Rs 5 and per km charge = Rs 10

Charge for 25 km = x + 25y = 5 + 250 = Rs 255

(v) A fraction becomes 9/11 , if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes 5/6. Find the fraction.

Solution:

Let the fraction be x/y.

According to the question,

$(x+2) /(y+2) = 9/11$

$11x + 22 = 9y + 18$

$11x – 9y = -4$ …………….. (1)

$(x+3) /(y+3) = 5/6$

$6x + 18 = 5y +15$

$6x – 5y = -3$ ………………. (2)

From (1), we get $x = (-4+9y)/11$ …………….. (3)

Substituting the value of x in (2), we get

$6(-4+9y)/11 -5y = -3$

$-24 + 54y – 55y = -33$

$-y = -9$

$y = 9$ …………………(4)

Substituting the valueofyin (3), we get

$x = (-4+9\times9)/11 = 7$

Hence the fraction is 7/9.

(vi) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Solutions:

Let the ageofJacoband his son be x and y respectively.

Accordingtothequestion,

$(x+5)=3(y+5)$

$x-3y=10$…………………………………….. (1)

$(x-5)=7(y-5)$

$x-7y=-30$………………………………………. (2)

From(1),wegetx$=3y + 10$ ……………………. (3)

Substitutingthevalueof x in (2), we get

$3y + 10 - 7y=-30$

$-4y = -40$

$y = 10$ ………………… (4)

Substituting the value of y in (3), we get

$x = 3 \times 10 + 10 = 40$

Hence, the present age of Jacob's and his son is 40 years and 10 years respectively.